Appendix wlb-00793 Herbert, J. A., Chakraborty, A., Naylor, L. W and D. G. Krementz, D. G. 2020. Habitat associations of wintering dabbling ducks in the Arkansas Mississippi Alluvial Valley: implications for waterfowl management beyond the mallard. – Wildlife Biology 2020: wlb.00793

## Appendix 1

## Statistical analysis

We denote  $\alpha$  parameters with a superscript to refers to the stage of the model in which it appears whereas; its subscript denotes its order in a partition of real line, increasing from left to right. The subscript starts from 0 and ends at *k* for a *k*-category response.

We first defined a presence/absence indicator  $y^{(a)} = 1[y>0]$ , where 1[B] is the standard notation for a binary random variable that is 1 if B is true and 0 otherwise. In a hierarchical setting, we link abundance data *y* to covariate data *x* through a latent variable *z* in the first stage as follows.

(A1) 
$$y_i^{(a)} = c \, \mathbb{1} \Big[ \alpha_c^{(1)} < z_i^{(1)} < \alpha_{c+1}^{(1)} \Big], c = 0, 1$$
$$z_i^{(1)} = \boldsymbol{x}_i^{\mathsf{T}} \boldsymbol{\beta}^{(1)} + \boldsymbol{\theta}_i^{(1)} + \boldsymbol{\varepsilon}_{1,i},$$

where  $y_i^{(a)}$  represents the indicator for presence/absence for grid cell *i*, *c* indexes categories, and the bracket notation represents the condition for  $y_i^{(a)} = 1$ . Specifically, the value of the continuous surface  $z_i^{(1)}$  must be greater than the lower boundary  $\alpha_c^{(1)}$  and less than the upper boundary  $\alpha_{c+1}^{(1)}$ with  $\alpha_0^{(1)} = -\infty$  and  $\alpha_2^{(1)} = \infty$ . The  $\alpha_c^{(1)}$  parameter denote the boundary points on a real line that determines the category of *y* from the continuous *z*. Consequently, we are only estimating one boundary parameter ( $\alpha_1^{(1)}$ ) in the first stage. Note that for cells not surveyed, we have no data on *y*. Furthermore,  $z_i^{(1)}$  represents a presence-absence surface,  $x_i^T \beta^{(1)}$  is the fixed effect mean for the presence/absence surface expressed as a linear combination of covariates,  $\theta_i^{(1)}$  is the spatial random effect, and ( $\epsilon_{1,i}$ ) is the pure error term. Thus, we are modeling presence/absence in the first stage.

In the second stage, we model the observed abundance category  $(y_i)$  given presence of mallards in grid cell i ( $y_i^{(a)} = 1$ ):

(A2) 
$$y_i | [y_i^{(a)} = 1] = (c+1) 1 \left[ \alpha_c^{(2)} < z_i^{(2)} < \alpha_{c+1}^{(2)} \right], c = 0, 1, \dots, k-1$$
$$z_i^{(2)} = x_i^{\mathrm{T}} \beta^{(2)} + \theta_i^{(2)} + \epsilon_{2,i}.$$

where  $y_i | [y_i^{(a)} = 1]$  represents the indicator for conditional abundance for grid cell *i*,  $z_i^{(2)}$  indicates a conditional abundance surface given presence  $\geq 1$ . The value of the conditional abundance surface  $z_i^{(2)}$  must be greater than the lower boundary  $\alpha_c^{(2)}$  and less than the upper boundary  $\alpha_{c+1}^{(2)}$ with  $\alpha_0^{(2)} = -\infty$  and  $\alpha_k^{(2)} = \infty$ . The  $\alpha_c^{(2)}$  parameters denote the boundary points on a real line that determines the category of *y* from the continuous *z*. Hence, the leftmost of these boundary points should be at  $-\infty$  and rightmost at  $+\infty$ , implying ( $\alpha_0^{(2)} = -\infty$  and  $\alpha_k^{(2)} = \infty$ ). The  $y_i$  and  $x_i$  denote the observed abundance category and the *p*-dimensional vector of available covariates.  $\beta^{(2)}$  is the covariate effects on the abundance surfaces. We do not need an intercept term in the regression model to ensure the parameters are identifiable.  $x_i^T \beta^{(1)}$  is the fixed effect mean for the conditional abundance surface expressed as a linear combination of covariates,  $\theta_i^{(2)}$  is the spatial random effect, and ( $\in_{2,i}$ ) is the pure error term. Thus, we are modeling only presence in the second stage. For each stage, the model has three parts: (1) a fixed effect mean expressed as a linear combination of covariates ( $x_i^T \beta^{(f)}$ ); (2) a random effect to capture the spatially correlated pattern present in the *z*surface ( $\theta_i^{(f)}$ ); and, (3) a pure error term accounting for residual variation ( $\in_{j,i}$ ).

Our dataset included mallard observations collected over multiple surveys and years, so we extended the above model into a spatio-temporal setting. We denoted a typical time point as  $(t_1, t_2)$  where  $t_1$  and  $t_2$  represent the year and survey, respectively. The notation  $y_i$   $(t_1, t_2)$  was the abundance category reported at cell  $A_i$  for year  $t_1$  and survey  $t_2$ . Similarly, we modified the notations for quantities in Eq. A1 and A2 to indicate their time-dependent characteristics. We focused on analyzing dependence between models at different points of time, anticipating temporal dependence across surveys as well as years. Mathematically, we expected y  $(t_1, t_2)$  to have temporal association with y  $(t_1-1, t_2)$ , y  $(t_1+1, t_2)$ , y  $(t_1, t_2-1)$  and y  $(t_1, t_2+1)$ . We denoted temporal dependence in the models for  $z^{(1)}$  and  $z^{(2)}$  through first-order autoregressive (AR) priors on  $\beta^{(1)}$  and  $\beta^{(2)}$ , respectively, as follows:

(Eq. A3) 
$$\beta^{(j)}(t_1, t_2) = \Gamma^{(j)}_{year} \beta^{(j)}(t_1 - 1, t_2) + \Gamma^{(j)}_{survey} \beta^{(j)}(t_1, t_2 - 1) + \eta^{(j)}(t_1, t_2), j = 1, 2.$$

The components of  $\Gamma_{year}$  and  $\Gamma_{survey}$  represented the AR coefficients for each of the *p* covariate effects across surveys and years, respectfully, in *x*, while  $\eta$  represented the pure error accounting for variation uncorrelated across time.

We modeled at two temporal scales, within-year (single year) and among-year (combined all

surveys). Our first objective was to estimate covariate effects on mallard distribution (stage 1) for each survey; covariate effects for the conditional abundance for each survey (stage 2); and, covariate-specific effects for temporal dependence across surveys (survey effect) and years (year effects). Our second objective was to produce spatial maps of random effects for each survey to explain how covariates performed among regions within the ARMAV. Our final objective was to develop spatial probabilities of mallard abundance across the ARMAV. To achieve this objective, we generated maps with the estimated categorical abundance probabilities throughout the ARMAV for each survey. Our first and second objectives follow directly from the analysis described in the equations above. We accomplished our final objective by expressing the categorical abundance probabilities as a function of model parameters. We predicted the probability that grid cell *i* had mallards present with:

(A4)  $P[y_i = 0] = \Phi(\alpha_1^{(1)} - x_i^{\mathrm{T}}\beta^{(1)} - \theta_i^{(1)}).$ 

For nonzero categories, with c = 0, 1, ..., k - 1,

$$P[y_i = c + 1] = \Phi\left(x_i^{\mathrm{T}}\beta^{(1)} + \theta_i^{(1)} - \alpha_1^{(1)}\right) \times \left[\Phi\left(\alpha_{c+1}^{(2)} - x_i^{\mathrm{T}}\beta^{(2)} - \theta_i^{(2)}\right) - \Phi\left(\alpha_c^{(2)} - x_i^{\mathrm{T}}\beta^{(2)} - \theta_i^{(2)}\right)\right],$$

Hence, given the posterior samples from parameters of  $\beta^{(j)}$  and  $\theta^{(j)}$  we constructed posterior maps of abundance probabilities.

We first evaluated the fit of our six candidate models with Bayesian  $\chi^2$  goodness of fit *p*-values. Specifically, our approach to evaluating model fit generated a test statistic for each MCMC iteration ( $R^B$ ) that asymptotically follows a  $\chi^2$  distribution under the assumption of model adequacy. We then simulated an equal length sample of independent  $\chi^2$ -random variables and determined the proportion of times a  $R^B$  draw is greater than the corresponding  $\chi^2$ -draw ( $P[R^B > \chi^2]$ ) to give our Bayesian *p*-value. In our analysis, we have 25 surveys each with a four-category response variable. Consequently, the degrees of freedom (*df*) of the  $\chi^2$  distribution was calculated as 25(4 - 1) = 75. The spatial random effects play an important role here as they have the ability to compensate for potential inadequacy of covariates present in a candidate model.

For potential concerns regarding the sample size being sufficient for the data analysis. Across 25 surveys, we have records from a total of 106 788 cells (there are 10 053 cells in the region). So, on an average, each survey contains records from around 4270 cells. The survey for Nov 2011 was of minimum sample size – presence-absence records were available from 3288 cells. The presence-absence distribution is as follows:

Survey No. cells with No. cells with

	absence	presence
		(in any category)
Nov, 2009	3502	181
Dec, 2009	3880	209
Early Jan,	4043	66
2010	4846	221
Late Jan,	4292	168
2010	4237	139
Nov, 2010	3604	125
Dec, 2010	3914	128
Early Jan,	3167	122
2011	4395	205
Late Jan,	4327	186
2011	4286	250
Nov, 2011	4075	174
Dec, 2011	4185	168
Early Jan,	4356	211
2012	4319	254
Late Jan,	4150	180
2012	4173	167
Nov, 2012	4246	78
Dec, 2012	4382	100
Early Jan,	4354	115
2013	4413	88
Late Jan,	4199	140
2013	4179	126
Nov, 2013	4368	199
Dec, 2013		
Early Jan,		
2014		
Nov, 2014		
Dec. 2014		
Early Jan,		
2015		
Nov, 2015		

Dec, 2015 Early Jan, 2016

For binary regression, the rule of thumb is to have at least 10 events/covariate where an "event" refers to a presence in our model (Peduzzi et al. 1996). The full model has 10 covariates. For most of the surveys, the number of cells with presence exceeds that ratio with a large enough margin. However, for three surveys (early-January surveys) suffer from small number of presences and fall significantly short of that.

However, the the hierarchical spatio-temporal combined survey-year modeling approach links all survey-specific covariate effects to each other through the covariate-specific parameters  $\Gamma_{survey}$  and  $\Gamma_{year}$ . As a result, the posterior distribution of  $\beta$  parameters of any one survey depends not only on the data specific to that survey, but, also on the surveys in the preceding and succeeding months of the same year as well as the same month of the preceding and succeeding years. Marginally, the posterior learning of any parameter gets contribution from entire dataset. From the above table, we have a total of 4000 cells with presence across 25 surveys. The total number of  $\beta$ parameters from all 25 surveys in the 1<sup>st</sup> stage is  $11 \times 25 = 275$ , resulting in a ratio of about 15 observations/covariate effect which is acceptable even under more conservative suggestions of 10-20 events/covariate as in Austin and Steyerberg (2017). All of these 4000 presence locations enter the 2<sup>nd</sup> stage model where again we deal with similar number of parameters. Since the two stages of the model are fitted separately, their posterior learning occurs independent of each other. Issues can arise if the analysis separately models each survey or move to an even larger set of covariates, small number of presences could be an issue. The usual solution for this problem is to regularize the coefficients (Pavlou et al. 2015) which, in a hierarchical setting, is equivalent to a lasso (Park and Casella 2008) or elastic net prior (Li and Lin 2010) on the  $\beta$  coefficients.

We also ranked covariate importance for each survey by dividing the posterior mean of each covariate effect by corresponding standard deviation (SD). Use of this measure is justified by the fact that, under large sample size, the mean and standard deviation of the posterior distribution of any regression coefficient approach its maximum likelihood estimation (MLE) and the asymptotic standard error of MLE, respectively (Ghosh and Ramamoorthi 2003). In that case, the ratio of empirical posterior mean to empirical posterior standard deviation can be viewed as the Bayesian equivalent to the t-statistic which is frequently used as a measure of strength of any covariate effect.

## References

- Austin, P.C. and Steyerberg, E.W. 2017. Events per variable (EPV) and the relative performance of different strategies for estimating the out-of-sample validity of logistic regression models. Statistical methods in medical research, 26(2), pp.796-808.
- Ghosh, J.K. and Ramamoorthi, R.V. 2003. Bayesian nonparametrics. Chapter 1.4. Springer Science & Business Media.
- Li, Q. and Lin, N., 2010. The Bayesian elastic net. Bayesian Analysis, 5(1), pp.151-170
- Pavlou, M., Ambler, G., Seaman, S. R., Guttmann, O., Elliott, P., King, M., & Omar, R. Z. (2015). How to develop a more accurate risk prediction model when there are few events. Bmj, 351, h3868.
- Park, T., & Casella, G. (2008). The Bayesian lasso. Journal of the American Statistical Association, 103(482), 681-686.
- Peduzzi P, Concato J, Kemper E, Holford TR, Feinstein AR (1996) A simulation study of the number of events per variable in logistic regression analysis. J Clin Epidemiology, 49(12), pp.1373-1379.





Figure A2.1 Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the November 2009 waterfowl survey. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.2** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the December 2009 waterfowl survey. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabblers than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.3** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the early-January 2010 waterfowl survey. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.4** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the late-January 2010 waterfowl survey, from the full within-year model. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.5** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the November 2010 waterfowl survey. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.6** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the December 2010 waterfowl survey. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.7** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the early-January 2011 waterfowl survey. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.8** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the late-January 2011 waterfowl survey, from the habitat within-year model. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.9** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the November 2011 waterfowl survey, from the habitat within-year model. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.10** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the December 2011 waterfowl survey, from the habitat within-year model. Scale of

 $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.11** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the early-January 2012 waterfowl survey, from the habitat within-year model. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.12** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the late-January 2012 waterfowl survey, from the habitat within-year model. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.13** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the November 2012 waterfowl survey. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.14** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the December 2012 waterfowl survey. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.15** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the early-January 2013 waterfowl survey, from the habitat within-year model. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.16** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the late-January 2013 waterfowl survey. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$ values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.17** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the November 2013 waterfowl survey, from the full within-year model. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.18** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the December 2013 waterfowl survey, from the full within-year model. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.19** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the early-January 2014 waterfowl survey. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.20** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the November 2014 waterfowl survey, from the habitat within-year model. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.21** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the December 2014 waterfowl survey. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.22** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the early-January 2015 waterfowl survey, from the habitat within-year model. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.23** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the November 2015 waterfowl survey. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.24** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the December 2015 waterfowl survey. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.



**Figure A2.25** Spatial random effects ( $\theta$ ) for the presence/absence of dabbling ducks across the Arkansas Mississippi Alluvial Valley (ARMAV) for the early-January 2016 waterfowl survey, from the full within-year model. Scale of  $\theta$  explains the performance of covariates used in the model. Positive  $\theta$  values represent cells with more dabbling ducks than expected and negative  $\theta$  values represent cells with less dabbling ducks than predicted. The entire surface of  $\theta$  values across the ARMAV equals 1.0. A correlated spatial pattern is shown, because a smooth gradient of  $\theta$  values exists, whereas a mixed gradient of  $\theta$  values would represent no spatial pattern present.