Wildlife Biology

WLB-00708

Fukasawa, K., Osada, Y. and Iijima, H. 2020. Is harvest size a valid indirect measure of abundance for evaluating the population size of game animals using harvest-based estimation? – Wildlife Biology 2020: wlb.00708

Appendix 1

Figure A1. Capture effort for data-generating scenarios.

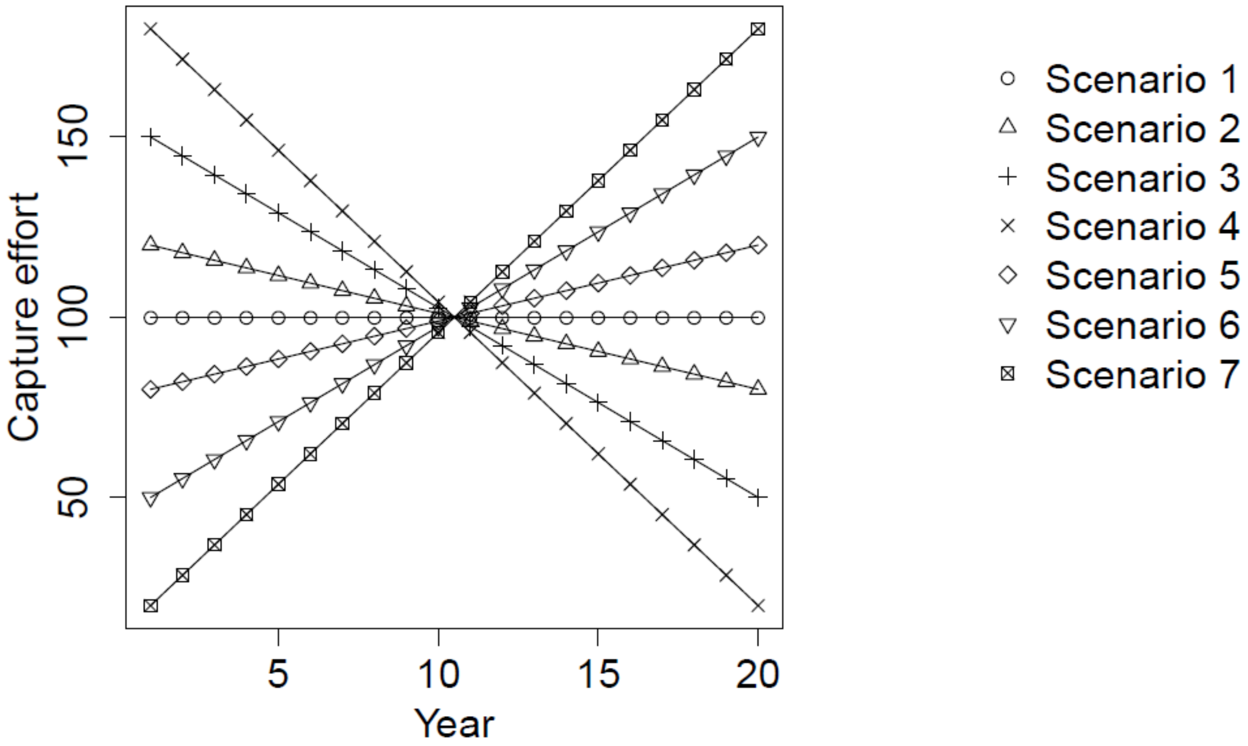


Figure A2. Harvest size generated under seven scenarios. Lines and error bars indicate mean and middle 90% range over 100 iterations, respectively.

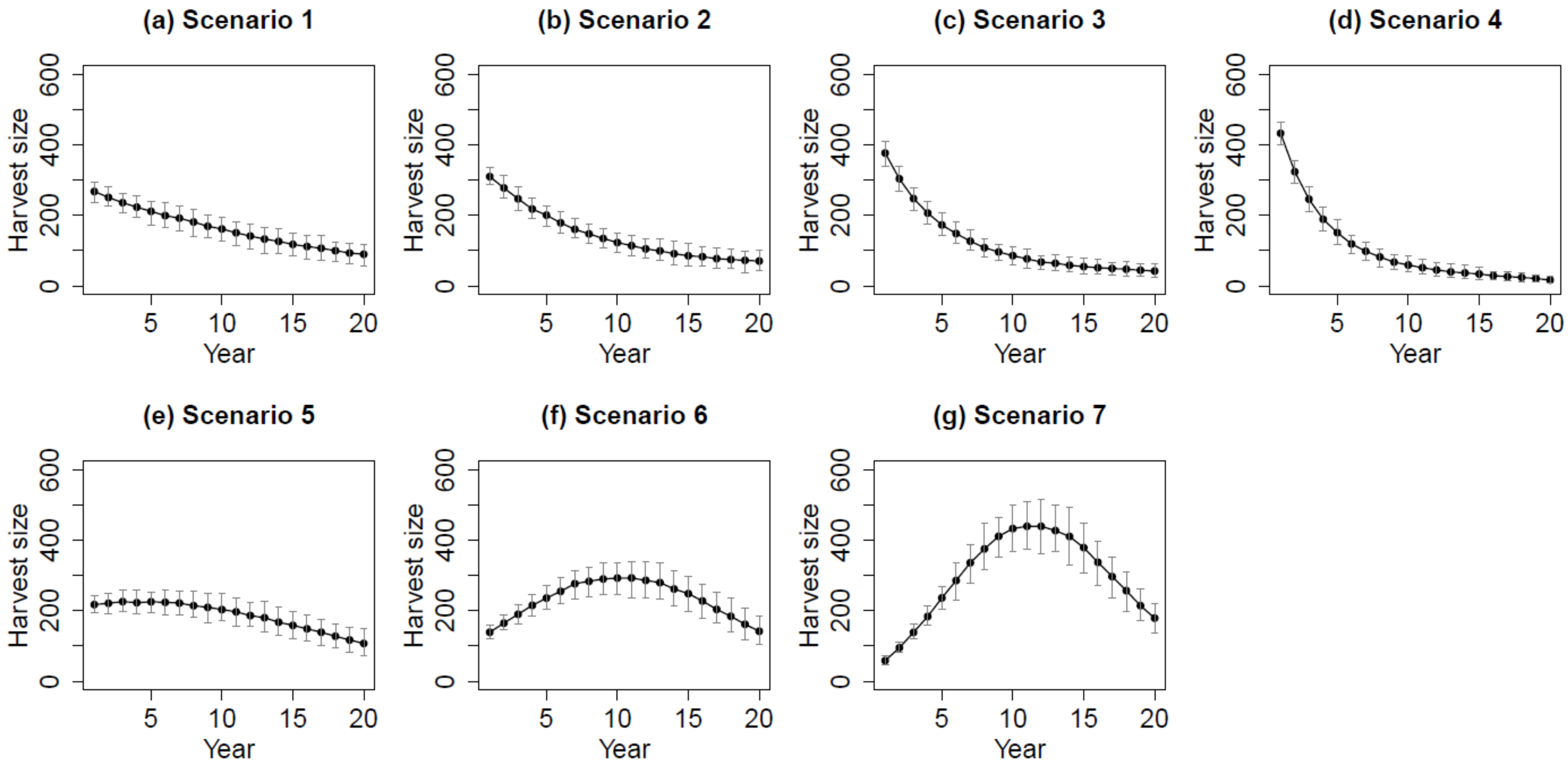
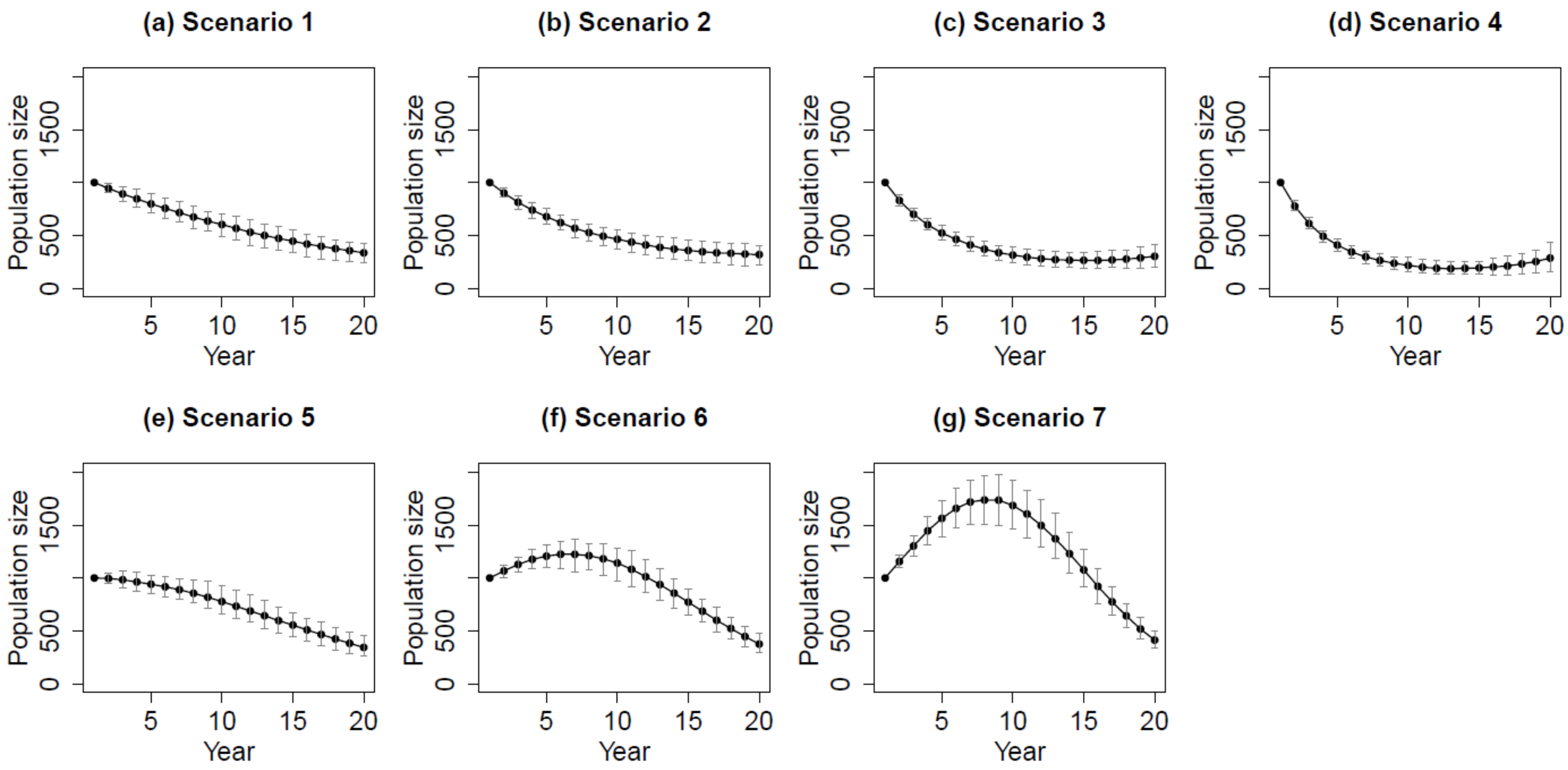


Figure A3. True population sizes generated under seven scenarios. Lines and error bars indicate means and middle 90% range over 100 iterations, respectively.



Appendix 2

Derivation of the marginal likelihood and smoothed distribution of state variables by recursive Bayesian filtering

Marginal likelihood, $p(C_{1:T} \mid \boldsymbol{\theta})$, can be factorized sequentially:

$$p(C_{1:T}|\boldsymbol{\theta}) = p(C_1|\boldsymbol{\theta}) \prod_{t=2}^{T} p(C_t|C_{1:t-1}, \boldsymbol{\theta}) \qquad (A1)$$

where T is the last time step t. We used vector notations for harvest size, $C_{1:t} = \{C_1, C_2, \ldots, C_t\}$. The observation probability of initial harvest size, $p(C_1 \mid \boldsymbol{\theta})$, is derived by integrating out S_1 from the product of the observation process distribution (Eq. 7) at $t = 1$, $p(C_1 \mid S_1, \boldsymbol{\theta})$, and prior distribution $p(S_1)$;

$$p(C_1|\boldsymbol{\theta}) = \sum_{S_1=0}^{S_{max}} p(C_1|S_1, \boldsymbol{\theta}) p(S_1).$$

where S_{max} is the maximum possible value for keeping the estimation procedure tractable and should be a sufficiently large value that does not affect the estimation.

The conditional probability, $p(C_t \mid C_{1:t-1}, \boldsymbol{\theta})$, in Eq. A1 is a probability that we observe C_t given the previous observations of $C_{1:t-1}$. It can be written as the expectation of the observation process distribution (Eq. 7), $p(C_t \mid S_t, \boldsymbol{\theta})$, over possible values of S_t given $C_{1:t-1}$;

$$p(C_t|C_{1:t-1}, \boldsymbol{\theta}) = \sum_{S_t=0}^{S_{max}} p(C_t|S_t, \boldsymbol{\theta})\, p(S_t|C_{1:t-1}, \boldsymbol{\theta}).$$

The conditional distribution of the latent state given previous observations, $p(S_t \mid C_{1:t-1}, \boldsymbol{\theta})$, is calculated by recursive filtering.

Recursive filtering is an online algorithm to calculate $p(S_t \mid C_{1:t-1}, \boldsymbol{\theta})$ for each t. In the process, the state prediction at t given the observation up to t-1 (prediction step) and updated posterior of state t incorporating the observation at t (update step) are operated recursively. The pseudocode of the algorithm is as follows.

```
Set prior of S_1

calculate p(S_1|C_1,θ) = p(C_1|S_1,θ)p(S_1) / Σ_{S_1=0}^{S_max} p(C_1|S_1,θ)p(S_1)            #update

for t = 2 to T do

      calculate p(S_t|C_{1:t-1},θ) = Σ_{S_{t-1}=0}^{S_max} p(S_t|S_{t-1},θ)p(S_{t-1}|C_{t-1},θ)    #prediction

      calculate p(S_t|C_{1:t},θ) = p(C_t|S_t,θ)p(S_t|C_{1:t-1},θ) / Σ_{S_t=0}^{S_max} p(C_t|S_t,θ)p(S_t|C_{1:t-1},θ)    #update

end do
```

Note that output of the previous update step, $p(S_{t-1} \mid C_{1:t-1}, \boldsymbol{\theta})$, is used for the prediction step at t. The state process distribution, $p(S_t \mid S_{t-1}, \boldsymbol{\theta})$, is defined in Eq. 6. The maximum a posteriori estimate of $\boldsymbol{\theta}$ is obtained by maximizing the log marginal likelihood, $\ln(p(C_{1:T} \mid \boldsymbol{\theta}))$.

After parameter estimates, $\hat{\boldsymbol{\theta}} = (\hat{r}, \hat{\varphi})$, are obtained, the smoothed distribution of S_t, $p(S_t \mid C_{1:T}, \hat{\boldsymbol{\theta}})$ and initial population size N_1, $p(N_1 \mid C_{1:T}, \hat{\boldsymbol{\theta}})$ are derived via forward-backward smoothing. In this smoothing procedure, the smoothed distribution at the last time step T, $p(S_T \mid C_{1:T}, \hat{\boldsymbol{\theta}})$, is calculated by recursive filtering, as described above, and smoothing at each time step t is executed by descending sequence (t = T-1, T-2, …, 1) according to the following equation:

$$p(S_t|C_{1:T},\hat{\boldsymbol{\theta}}) = p(S_t|C_{1:t},\hat{\boldsymbol{\theta}}) \sum_{S_{t+1}=0}^{S_{max}} \frac{p(S_{t+1}|C_{1:T},\hat{\boldsymbol{\theta}})p(S_{t+1}|S_t,\hat{\boldsymbol{\theta}})}{p(S_{t+1}|C_{1:t},\hat{\boldsymbol{\theta}})}.$$

Note that $p(S_t \mid C_{1:t}, \hat{\boldsymbol{\theta}})$ and $p(S_{t+1} \mid C_{1:t}, \hat{\boldsymbol{\theta}})$ are outputs of the update step and prediction step of recursive filtering, respectively. $p(S_{t+1} \mid C_{1:T}, \hat{\boldsymbol{\theta}})$ is the smoothed distribution of S_{t+1} calculated in the previous step of smoothing. $p(N_1 \mid C_{1:T}, \hat{\boldsymbol{\theta}})$ is obtained from the smoothed distribution of S_1 as follows:

$$p(N_1|C_{1:T},\hat{\boldsymbol{\theta}}) = \sum_{S_1=0}^{S_{max}} \frac{p(S_1|C_{1:T},\hat{\boldsymbol{\theta}})p(S_1|N_1,\hat{\boldsymbol{\theta}})}{p(S_1)}.$$

Note that $p(S_1 \mid N_1, \hat{\boldsymbol{\theta}})$ is identical to equation 1. The point estimate of N_1, $\hat{N}_1$, is defined as the expected value of N_1 as follows:

$$\hat{N}_1 = \sum_{N_1=0}^{S_{max}} N_1\, p(N_1|C_{1:T},\hat{\boldsymbol{\theta}}).$$