Wildlife Biology **WLB-00094**

Korner-Nievergelt, F., Behr, O., Brinkmann, R., Etterson, M. A., Huso, M. M. P, Dalthorp, D., Korner-Nievergelt, P., Roth, T. and Niermann, I. 2015. Mortality estimation from carcass searches using the R-package carcass. – Wildlife Biology doi: 10.2981/wlb.00094

Supplementary information 1

Derivation of the formula `pkorner` allowing for decreasing searcher efficiency *f*

The formula is already published and tested in Korner-Nievergelt et al. (2011) but without a description of how we obtained it. The formula includes k, which is the factor by which the searcher efficiency is multiplied at each subsequent search for a given cohort (here: animals that were killed during the same inter-search interval).

We first note the expected number of carcasses remaining on the ground (in the search area) for 1 to d days with m carcasses falling into the search area per day (s = persistence probability).

day	proportion of carcasses remaining
1	$m(s)$
2	$m(s + s^2)$
3	$m(s + s^2 + s^3)$
...	...
d	$m(s + s^2 + s^3 + ... + s^d)$; the sum (in brackets) up to $s^d = s(1\text{-}sd)/(1\text{-}s)$ = A

The number of carcasses that are found during the first search is (f = searcher efficiency):

first search (after d days)	c_1:	mAf
	number missed:	$mA(1 - f)$

At the second search, we have the carcasses that were overlooked during the first search, if they have not disappeared in the meantime (i.e. during the d days between the first and second search): $mA(1\text{-}f)s^d$ plus the new ones that have not disappeared, i.e. mA. The detection probability is fk for the old carcasses and f for the new carcasses:

second search	c_2:	$mA(1-f)s^d fk$	$+ mAf$
	number missed:	$\underbrace{mA(1-f)sd(1-fk)}_{\alpha}$	$\underbrace{+ mA(1-f)}_{\beta}$

Before the next search, the probability for an animal that had died to be still there is the α-term times s^d (carcasses from the first search interval that have not disappeared until the second search, not found during the second search, and again not disappeared until the third search), plus the β-term times s^d, plus the new carcasses, mA:

$$mA(1-f)s^d(1-fk)s^d + mA(1-f)s^d + mAs^d$$

The detection probability is fk^2 for the first summand, fk for the second and f for the third. Factoring out mAf, we get the following summands for the expected number of carcasses found c during n searches at d-day intervals (each mAf[..]-summand corresponding to one of the n searches):

$$mAf[1] +$$

$$mAf[1+(1-f)s^d k] +$$

$$mAf[1+(1-f)s^d k+(1-f)s^d (1-fk)s^d k^2] +$$

$$mAf[1+(1-f)s^d k+(1-f)s^d (1-fk)s^d k^2 +(1-f)s^d (1-fk)s^d (1-fk^2)s^d k^3]+$$

The parts in the brackets can be described with sums and products doing some careful thinking about the indexing. Dividing the whole sum by the total number of carcasses that have fallen into the search area during the study period (mnd) we get the proportion of these carcasses that are found (detection probability p):

$$p = \frac{Af[n + \sum_{x=2}^{n} \sum_{j=1}^{x-1} k^{(x-j)} s^{(x-j)d} \prod_{i=0}^{x-j-1} (1 - fk^i)]}{nd}$$

Derivation of the formula `pkorner` allowing for age-dependent persistence probabilities

To allow for age-dependent persistence probabilities, we do not use a compact mathematical formula. Instead, we calculate for each cohort of carcasses (here: animals that have died during the same time interval, normally one day) and for each subsequent day the average proportion of remaining carcasses on the ground given the age-specific proportion of persisting carcasses, the searcher efficiency and the search interval (carcasses that are found during a search are removed from the search area). Similarly, we calculate for each cohort and day the proportion of carcasses that were found. At the end, we use the average proportion of found carcasses over all cohorts and days.

References

Korner-Nievergelt, F. et al. 2011. A new method to determine bird and bat fatality at wind energy turbines. – J. Wildl. Biol. 17: 350–363.

Supplementary information 2

Fränzi Korner-Nievergelt, Oliver Behr, Robert Brinkmann, Matthew A. Etterson,
Manuela M. P. Huso, Dan Dalthorp, Pius Korner-Nievergelt, Tobias Roth and Ivo Niermann:
“Mortality estimation from carcass searches using the R-package carcass – a tutorial” – Wildlife Biology 000: 001–014.

Mathematical Proof of the equality of Etterson (2013) Eq. 15 and Korner-Nievergelt et al. (2011:352) Estimator.

Etterson (2013) Eq. 15 states:

$$N_{fe} = \lambda p_d \frac{q_r\left(1-q_r^{\bar{d}}\right)}{\left(1-q_r\right)\left(1-q_d q_r^{\bar{d}}\right)}\left(n - q_d q_r^{\bar{d}}\left(\frac{1-\left(q_d q_r^{\bar{d}}\right)^n}{1-q_d q_r^{\bar{d}}}\right)\right)$$, where:

N_{fe} = number of carcasses found
p_d = per search detection probability (q_d = 1 – p_d = prob. of non-detection per search)
p_r = scavenging probability per day (q_r = 1 – p_r = persistence probability per day)
d = fixed interval between searches
n = number of searches
λ = number of animals killed per day

The Korner-Nievergelt et al. (2011:352) equation states:

$$C = \overline{m} f\left(s\frac{1-s^d}{1-s}\right)\left(\sum_{i=0}^{n-1}\left(n-i\right)\left(\left(1-f\right)s^d\right)^i\right)$$, where:

C = number of carcasses found
f = per search detection probability
s = daily persistence probability
d = fixed interval between searches
n = number of searches
$\overline{m}$ = number of animals killed per day

Therefore the following relationships between Etterson’s (2013) parameters and the Korner-Nievergelt et al. (2011) parameters is:

N_{fe} = C
$p_d = f$ (and $q_d = 1-f$)
$p_r = 1-s$ (and $s = q_r$)
$d = d$
$n = n$
$\lambda = \overline{m}$

Substituting the Korner-Nievergelt et al. (2011) symbols into Etterson's (2013) equation gives:

$$C_E = \overline{m}f \frac{s(1-s^d)}{(1-s)(1-(1-f)s^d)}\left(n-(1-f)s^d\left(\frac{1-((1-f)s^d)^n}{1-(1-f)s^d}\right)\right)$$, where the subscript E

indicates that this formulation comes from the Etterson (2013) Equation. The subscript KN will be used to indicate the Korner-Nievergelt et al. (2011:352) equation. The reformulated equation from Etterson (2013) can be rearranged to:

$$C_E = \overline{m}fs\frac{(1-s^d)}{(1-s)}\frac{1}{(1-(1-f)s^d)}\left(n-(1-f)s^d\left(\frac{1-((1-f)s^d)^n}{1-(1-f)s^d}\right)\right).$$

Compare this now with the Korner-Nievergelt et al. (2011) equation:

$$C_{KN} = \overline{m}f\left(s\frac{1-s^d}{1-s}\right)\left(\sum_{i=0}^{n-1}(n-i)((1-f)s^d)^i\right)$$

Clearly we can divide both equations by $\overline{m}f\left(s\frac{1-s^d}{1-s}\right)$ to get:

$$E = \frac{1}{(1-(1-f)s^d)}\left(n-(1-f)s^d\left(\frac{1-((1-f)s^d)^n}{1-(1-f)s^d}\right)\right)$$, and:

$$KN = \left(\sum_{i=0}^{n-1}(n-i)((1-f)s^d)^i\right)$$, where the symbols *E* and *KN* denote the expressions

originating from the Etterson (2013) equation and the Korner-Nievergelt et al. (2011) equation respectively. Therefore it is sufficient to show that *E*=*KN* to show that the two equations are equivalent. To simplify the equations, let's define the term *Q*, as $Q = (1-f)s^d$ and substitute into both equations. Then:

$$E = \frac{1}{(1-Q)}\left(n-Q\left(\frac{1-Q^n}{1-Q}\right)\right) \text{ and } KN = \left(\sum_{i=0}^{n-1}(n-i)Q^i\right)$$

The proof follows below. The strategy is to derive *E* by expanding *KN* and deriving algebraic expressions for the summation terms. First note that we can expand *KN* as follows:

$$KN = \left(\sum_{i=0}^{n-1}(n-i)Q^i\right) = \sum_{i=0}^{n-1}nQ^i - \sum_{i=0}^{n-1}iQ^i$$

The first term in *KN* (denoted below as KN_1, where $KN = KN_1 - KN_2$) is easy to expand (see derivation in **Power Series 1** at end of this proof):

$$KN_1 = \sum_{i=0}^{n-1}nQ^i = n\sum_{i=0}^{n-1}Q^i = n\frac{1-Q^n}{1-Q}$$

The second term can also be expanded using the power rule for derivatives. First we must note that:

$$KN_2 = \sum_{i=0}^{n-1} iQ^i = \sum_{i=1}^{n-1} iQ^i$$

In this case (see derivation in **Power Series 2** at the end of this proof):

$$KN_2 = \sum_{i=1}^{n-1} iQ^i = Q\sum_{i=1}^{n-1} iQ^{i-1} = Q\sum_{i=1}^{n-1} \frac{d}{dQ}\left(Q^i\right) = Q\frac{d}{dQ}\left(\sum_{i=1}^{n-1} Q^i\right) = Q\frac{d}{dQ}\left(\frac{Q\left(1-Q^{n-1}\right)}{1-Q}\right)$$

Using the quotient rule we can now solve this as:

$$KN_2 = Q\frac{d}{dQ}\left(\frac{Q\left(1-Q^{n-1}\right)}{1-Q}\right) =$$

$$= Q\frac{(1-Q)\frac{d}{dQ}\left(Q\left(1-Q^{n-1}\right)\right) - Q\left(1-Q^{n-1}\right)(-1)}{(1-Q)^2} =$$

$$= Q\frac{(1-Q)\left[\left(Q\left(-(n-1)Q^{n-2}\right)\right) + \left(1-Q^{n-1}\right)\right] - Q\left(1-Q^{n-1}\right)(-1)}{(1-Q)^2} =$$

$$= Q\frac{(1-Q)\left[\left(-(n-1)Q^{n-1}\right) + 1 - Q^{n-1}\right] + Q\left(1-Q^{n-1}\right)}{(1-Q)^2} =$$

$$= Q\frac{(1-Q)\left[1-nQ^{n-1}\right] + Q\left(1-Q^{n-1}\right)}{(1-Q)^2} =$$

$$= Q\frac{(1-Q)\left[1-nQ^{n-1}\right]}{(1-Q)^2} + \frac{Q^2\left(1-Q^{n-1}\right)}{(1-Q)^2} =$$

$$KN_2 = Q\frac{\left(1-nQ^{n-1}\right)}{(1-Q)} + \frac{Q^2\left(1-Q^{n-1}\right)}{(1-Q)^2}$$

Now we can put the expanded first and second terms of *KN* together to get:

$$KN = KN_1 - KN_2 = n\frac{1-Q^n}{1-Q} - Q\frac{\left(1-nQ^{n-1}\right)}{(1-Q)} - \frac{Q^2\left(1-Q^{n-1}\right)}{(1-Q)^2}$$

Now factor out 1/(1 – Q):

$$KN = \frac{1}{1-Q}\left(n\left(1-Q^n\right) - Q\left(1-nQ^{n-1}\right) - \frac{Q^2\left(1-Q^{n-1}\right)}{(1-Q)}\right)$$

Now rearrange:

$$KN = \frac{1}{1-Q}\left(n - nQ^n - Q + nQ^n - \frac{Q^2\left(1-Q^{n-1}\right)}{(1-Q)}\right) =$$

$$= \frac{1}{1-Q}\left(n - Q - \frac{Q^2\left(1-Q^{n-1}\right)}{(1-Q)}\right) = \frac{1}{1-Q}\left(n - Q - Q\frac{\left(Q-Q^n\right)}{(1-Q)}\right) =$$

$$= \frac{1}{1-Q}\left(n - Q\left(1 + \frac{\left(Q-Q^n\right)}{(1-Q)}\right)\right) =$$

$$= \frac{1}{1-Q}\left(n - Q\left(\frac{1-Q}{1-Q} + \frac{Q-Q^n}{1-Q}\right)\right) =$$

$$KN = \frac{1}{1-Q}\left(n - Q\left(\frac{1-Q^n}{1-Q}\right)\right) = E$$

The above expression is *E*, as required. Therefore the two equations are equivalent.

Power Series 1

Let: $KN_1 = nx$, where:

$$x = \sum_{i=0}^{n-1} Q^i = 1 + Q + Q^2 + \ldots + Q^{n-1}.$$

Below is the expansion of *x*:

$\frac{x-1}{Q} + Q^{n-1} = x$, and

$\frac{x-1}{Q} + Q^{n-1} = x$, in which case

$x - 1 + Q^n = xQ$, in which case

$x - xQ = 1 - Q^n$, and

$x(1-Q) = 1 - Q^n$

$$x = \frac{1-Q^n}{1-Q}$$

Therefore:

$$KN_1 = n\frac{1-Q^n}{1-Q}$$

Power Series 2

Let: $KN_2 = \sum_{i=1}^{n-1} iQ^i = Q\sum_{i=1}^{n-1} iQ^{i-1} = Q\sum_{i=1}^{n-1}\frac{d}{dQ}\left(Q^i\right) = Q\frac{d}{dQ}\left(\sum_{i=1}^{n-1} Q^i\right)$

Then: $KN_2 = Q\frac{d}{dQ}(x)$, where:

$$x = \sum_{i=1}^{n-1} Q^i = Q + Q^2 + ... + Q^{n-1}$$

Below is the expansion of *x*:

$\frac{x-Q}{Q} + Q^{n-1} = x$, in which case:

$x - Q + Q^n = xQ$, and

$x - xQ = Q - Q^n$, and

$x(1-Q) = Q(1-Q^{n-1})$, in which case:

$x = \frac{Q(1-Q^{n-1})}{(1-Q)}$, and finally:

$$KN_2 = Q\frac{d}{dQ}\left(\frac{Q(1-Q^{n-1})}{1-Q}\right)$$